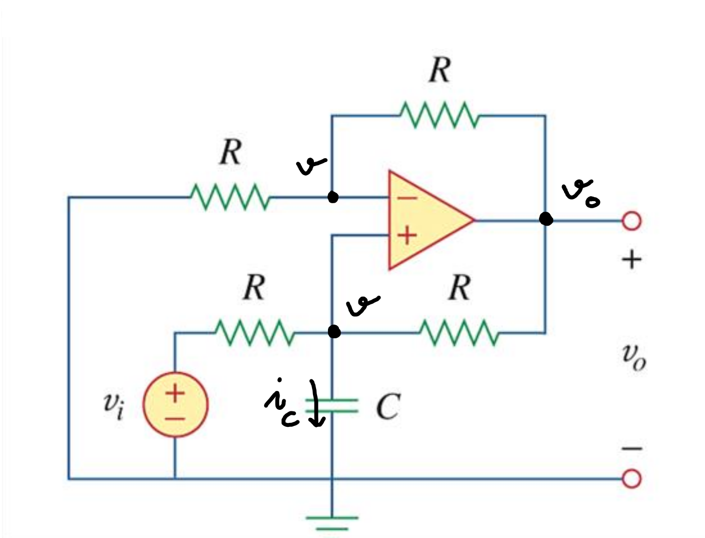
Q1 [Alexander and Sadiku, 2009, Q6.73]

Monday, August 12, 2013 9:24 PM



For ideal op amp: $v_- = v_+ \equiv v$

$i_- = i_+ = 0$

At the negative input terminal:

$$\frac{v-0}{R} + \frac{v-v_o}{R} = 0$$

$$2v = v_o \quad \Rightarrow v = \frac{v_o}{2}$$

At the positive input terminal:

$$\frac{v-v_i}{R} + i_c + \frac{v-v_o}{R} = 0$$

$$i_c = C\frac{dv_c}{dt} = C\frac{d}{dt}(v-0) = C\frac{d}{dt}\left(\frac{v_o}{2}\right) = \frac{C}{2}\frac{dv_o}{dt}$$

$$\cancel{\frac{v_o}{2}} - v_i + \frac{RC}{2}\frac{dv_o}{dt} + \cancel{\frac{v_o}{2}} - \cancel{v_o} = 0$$

$$\frac{dv_o}{dt} = \frac{2}{RC}v_i$$

$$\boxed{v_o(t) = v_o(t_0) + \frac{2}{RC}\int_{t_0}^{t} v_i(\alpha)\,d\alpha}$$

Q2 [Alexander and Sadiku, 2009, Ex 9.1]

Tuesday, August 20, 2013 7:35 PM

Recall that for a sinusoid in standard form

$$x(t) = X_m \cos(\omega t + \phi)$$

The amplitude is X_m

The phase is ϕ.

The angular frequency is ω.

The period is $T = \frac{2\pi}{\omega}$.

The frequency is $f = \frac{\omega}{2\pi} = \frac{1}{T}$.

Here, $v(t) = 12\cos(50t + \phi)$. Therefore,

The amplitude is 12.

The phase is 10°.

The angular frequency is $\omega = 50$.

The frequency is $\frac{\omega}{2\pi} = \frac{50}{2\pi} = \frac{25}{\pi} \approx 7.958$.

The period is $\frac{1}{\text{freq.}} = \frac{1}{25/\pi} = \frac{\pi}{25} \approx 0.1257$.

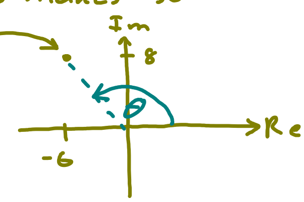
Q3&4 Working with Complex Numbers

Thursday, April 2, 2015 2:22 PM

The next two questions are here to give you some warm-up exercise for the computation that you will encounter throughout the next couple chapters. You will need to be able to work with complex numbers and many of the calculations will require the use of a calculator. Since all of the work is done on the calculator, only the answers are provided here.

③ a) $-6+8j = 10\angle 127^\circ$

Getting an angle $\in (90^\circ, 180^\circ)$ makes sense because $-6+8j$ is here



This can be found by hand via $\sqrt{(-6)^2+8^2} = 10$

b) $\dfrac{50\angle -30^\circ}{10j+5-2j} = \dfrac{50\angle -30^\circ}{5+8j} \approx 5.3\angle -88^\circ$

This can be found by hand via $\dfrac{50}{\sqrt{5^2+8^2}} = \dfrac{50}{\sqrt{89}}$

(Recall that $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$.)

④ a) $-10j + \dfrac{(3-2j)\times(8+10j)}{(3-2j)+(8+10j)} \approx 3.22 - 11.1j$

Alternatively, one can try to work on this part by hand:

$$\frac{(3-2j)\times(8+10j)}{(3-2j)+(8+10j)} = \frac{44+14j}{11+8j} = \frac{(44+14j)(11-8j)}{(11-8j)(11-8j)} = \frac{596-198j}{11^2+8^2}$$

$$= \frac{596}{185} - \frac{198}{185}j$$

Finally, after adding "$-10j$" to the result, we have

$$\frac{596}{185} - \left(\frac{198}{185}+10\right)j = \frac{596}{185} - \frac{2048}{185}j$$

b) $(20\angle 15^\circ)\times \dfrac{100j}{60+100j} \approx 17.15\angle 15.96^\circ$

Alternatively, we can first convert every terms to polar form:

$100j = 100\angle 90^\circ$

$60+100j = 20\sqrt{34}\angle 59^\circ$

Therefore, $\sqrt{60^2+100^2} = 10\sqrt{36+100} = 20\sqrt{9+25} = 20\sqrt{34}$

Therefore, $\sqrt{60^2+100^2} = 10\sqrt{36+100} = 20\sqrt{9+25} = 20\sqrt{34}$

$$(20\angle{-15^\circ}) \times \frac{100j}{60+100j} = \frac{\cancel{20}\times 100}{\cancel{20}\sqrt{34}} \angle \underbrace{(-15^\circ + 90^\circ - 59^\circ)}_{16^\circ}$$

Q5 Solving Equations Involving Complex-Valued Unknowns

Thursday, April 02, 2015 2:55 PM

We have three unknown variables (phasors) here: $\vec{I}_1$, $\vec{I}_2$, and $\vec{I}_3$.

However, because $\vec{I}_3 = \vec{I}_s = 5$, we actually have only two unknowns.

So, we will try to reorganize the remaining two equations so that $\vec{I}_1$ and $\vec{I}_2$ are on the LHS and everything else are on the RHS.

$$-\vec{I}_1\vec{Z}_3 - (\vec{I}_1 - \vec{I}_3)\vec{Z}_2 - (\vec{I}_1 - \vec{I}_2)\vec{Z}_4 = 0$$

$$\underbrace{(-\vec{Z}_3 - \vec{Z}_2 - \vec{Z}_4)}_{\substack{-8-10j-(-2j) \\ =-8-8j}}\vec{I}_1 + \underbrace{\vec{Z}_4}_{-2j}\vec{I}_2 = \underbrace{-\underbrace{\vec{I}_3}_{5}\underbrace{\vec{Z}_2}_{10j}}_{-50j}$$

$$(-8-8j)\vec{I}_1 + (-2j)\vec{I}_2 = -50j \quad \times -\tfrac{1}{2}$$

$$(+4+4j)\vec{I}_1 + (+j)\vec{I}_2 = +25j \quad \cdots (1)$$

$$-(\vec{I}_2 - \vec{I}_1)\vec{Z}_4 - (\vec{I}_2 - \vec{I}_3)\vec{Z}_1 - \vec{I}_2\vec{Z}_5 - \vec{V}_s = 0$$

$$\underbrace{\vec{Z}_4}_{-2j}\vec{I}_1 + \underbrace{(-\vec{Z}_4 - \vec{Z}_1 - \vec{Z}_5)}_{\substack{2j+2j-4 \\ =-4+4j}}\vec{I}_2 = \underbrace{\vec{V}_s - \vec{I}_3\vec{Z}_1}_{\substack{20\angle 90^\circ - 5(-2j) \\ =20j+10j=30j}}$$

$$(-2j)\vec{I}_1 + (-4+4j)\vec{I}_2 = 30j$$

$$(-j)\vec{I}_1 + (-2+2j)\vec{I}_2 = 15j \quad \times \frac{1}{-j} = j$$

$$\vec{I}_1 + (-2-2j)\vec{I}_2 = -15$$

$$\vec{I}_1 = -15 + (2+2j)\vec{I}_2$$

substitute this $\vec{I}_1$ into (1) to get $(4+4j)\left(-15 + (2+2j)\vec{I}_2\right) + j\vec{I}_2 = 25j$

which gives $\vec{I}_2 = \dfrac{25j + 15(4+4j)}{(4+4j)(2+2j)+j} = \dfrac{60+85j}{17j} = 5 - \dfrac{60}{17}j$

$$\approx 5 - 3.53j \approx 6.12\angle -35.2^\circ$$

Q6 Sinusoids to Phasors

Tuesday, August 20, 2013 8:01 PM

When working with standard form, remember two useful facts:

(1) $\sin x = \cos(x - 90^\circ)$

(2) $-\cos x = \cos(x \pm 180)$

Pick one that gives the net phase $\in [-180^\circ, 180^\circ]$

a) $v(t) = 120 \sin(10t - 50^\circ) = 120 \cos(10t - 50^\circ \underbrace{- 90^\circ}_{\sin \to \cos}) = 120 \cos(10t - 140^\circ)$

$\Updownarrow$

$\vec{V} = 120 \angle -140^\circ$ V

b) $i(t) = -60 \cos(30t + 10^\circ) = 60 \cos(30t + 10^\circ \underbrace{- 180^\circ}_{-\cos \to \cos}) = 60 \cos(30t - 170^\circ)$

$\Updownarrow$

$\vec{I} = 60 \angle -170^\circ$ mA

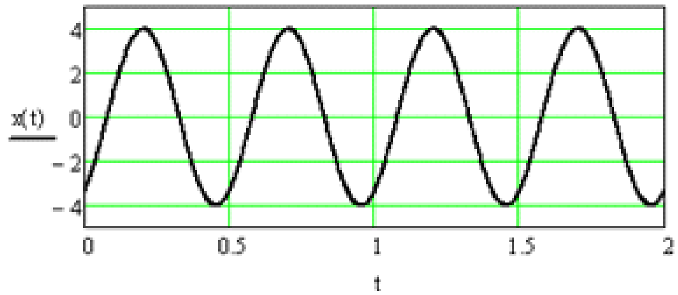
c) $i(t) = -8 \sin(10t + 70^\circ) = 8 \cos(10t + 70^\circ \underbrace{- 90^\circ}_{\sin \to \cos} \underbrace{+ 180^\circ}_{-\cos \to \cos}) = 8 \cos(10t + 160^\circ)$

$\Updownarrow$

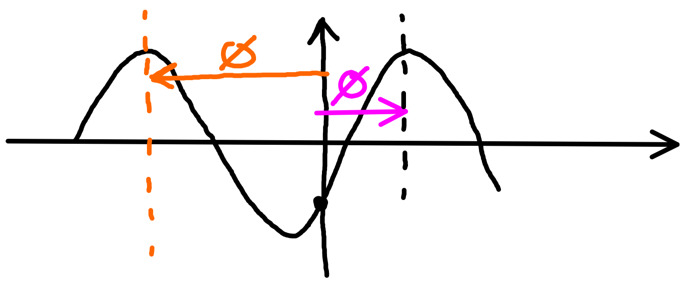
$\vec{I} = 8 \angle 160^\circ$ mA

Q7 Finding phase of a sinusoid

Tuesday, August 20, 2013 8:37 PM



First we observe that the wave form is the same as cosine function shifted to the right by ϕ where ϕ is between $90°$ and $180°$



Shifting to the right means ϕ is negative.
So,

$$-180° < \phi < -90°$$

Equivalently, the graph is also the cosine function shifted to the left by ϕ where

$$180° < \phi < 270°$$

Now,
from the general form of sinusoidal waveform

$$x(t) = A\cos(\omega t + \phi)$$

From the plot, we have $A = 4$.

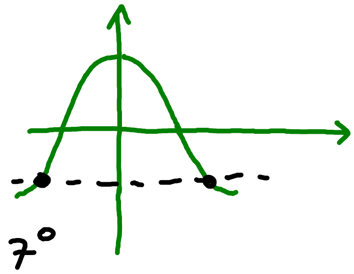
$$x(0) = 4\cos(\phi)$$

$$\shortparallel$$

$$-3.356$$

$$= -3.356$$

$$\cos \phi = -\frac{3.356}{4} \approx -0.839$$



Two solutions: $\phi = 147°$ and $-147°$

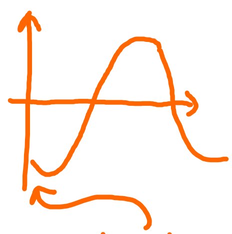
Because ϕ must be between $-180°$ and $-90°$, we know that $\phi = -147°$.

Therefore,

$$\vec{X} = 4 \angle -147°$$

Note that $\phi = 147°$ will give different graph.

Try it! You will get



which start at the wrong position.